Testing for Co-integration and nonlinear adjustment in a Smooth Transition Error Correction Model

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First Draft: June 2009

Abstract

This paper introduces a testing procedure for cointegration and nonlinear adjustment in a smooth transition vector error correction model. To overcome the unidentified parameters problem under the null of no-cointegration, a sup-type Wald test statistic is proposed where the Wald statistic is optimized over an asymptotically well-defined parameter space. The asymptotic distribution of the test statistic ($supW$) is shown to be nonstandard but nuisance parameter-free and hence critical values are obtained by simulations. Therefore the suggested procedure does not require bootstrapping and can be applied easily in practice. The finite sample size and power properties of the test is examined and compared to some alternatives. Simulations show that the proposed test outperforms the alternatives in small sample sizes both in terms of size and power. In application to the exchange rate-monetary fundamentals relationships, our test is able to find cointegration, while the alternatives fail to do so.

Key Words: Cointegration, Smooth Transition Vector Error Correction, Nonlinearity, Monetary Model.

JEL Classification: C12, C22, F41 (Time Series Econometrics).

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1 Introduction

In this paper, we propose a testing procedure for no cointegration (or no linear and nonlinear adjustment) in the context of a smooth transition vector error correction model (ST-VECM) where the cointegration vector is both assumed to be known (and hence pre-specified) and unknown (and hence pre-estimated). Both symmetric and asymmetric type nonlinear adjustments are considered by using exponential and logistic transition variables under various potential transition variables including the lagged deviations from the long-run equilibrium and lagged levels of an exogenous variable.

There are several cases where economic and financial theory suggest that variables are cointegrated (with in some cases known cointegrating vectors) and adjustment to the implied long-run relationships may involve nonlinearities. For example, theories of international tarde suggest long-run purchasing power parity, so that exchange rates should be matched by countries' relative price level. The several versions of neoclassical growth models imply that per-capita output and investment should grow in a balanced way so that any stochastic change in say investment should be matched by corresponding change in output. Models of exchange rate determination suggests that exchange rates and economic fundamentals including relative money supplies and output should move proportionally in the long run. Asset pricing models imply that spot and forward prices, or stock prices and dividends should move proportionally in the long-run.

More recent work also suggests that the adjustment mechanism to these long run relationships may have nonlinear dynamics. Theoretical work by Dumas (1992), Sercu, et al., (1995), Kilian and Taylor, Lyons (2001) and Baldwin (1988) among many others suggest that the relationship between exchange rates and various economic fundamentals such as relative prices, monetary fundamentals or interest rate differentials can involve nonlinear adjustment dynamics in the presence of transportation or transactions costs in trade, limits to speculation and hysteresis type behavior in exchange rate and commodity markets. Similarly work by De Long et al., (1990), Campbell and Kyle (1993) and Hong and Stain (1999) among many others imply that the relationship between asset prices and fundamentals (such as dividends) should be characterized by nonlinearities in the presence of heterogeneity in agents's expectations, institutional rigidities and transactions costs.

Threshold and smooth transition cointegration models have become popular in applied economic and financial work over the past decade. Examples include among many others, Swanson (1998), Rothman, Franses and Van Dijk (2001), Chen and Wu (2005) who estimated various nonlinear Vector Error Correction Models (VECM) models by employing Luukonen et al., (1988) type of tests for nonlinearities which were developed in stationary univariate setting. Econometric research that investigates extensions of the cointegration theory initiated by Granger (1981) and Engle and Granger (1987) to smooth transition settings has moved in two main directions. One direction has focused on modeling and testing nonlinear adjustment in deviations from (linear) long-run equilibrium relations. Examples of this approach include Balke and Fomby (1997), Enders and Engle (1998), Hansen and Seo (2002), and Bec and Rahbek (2004), Seo (2004) and Kapetanios et al., (2006). Another direction of research has involved modeling and testing nonlinearity in the cointegrating relations or time series. Examples of this line of research include Park and Phillips (1999, 2001), Chang, Park, and Phillips (2001), Saikkonen and Choi (2004), Choi and Saikkonen (2004 and 2005), and Gonzalo and Pitarakis (2006), Caner and Hansen (2001), Kapetanios et al. (2003), Park and Shintani (2005) and Kılıç (2009).

Within the first line of research, studies by Balke and Fomby (1997), Enders and Engle (1998), Hansen and Seo (2002), Bec and Rahbek (2004) have focused on testing abrupt adjustment mechanisms within threshold vector error correction models where linearity is assumed for the cointegrating relations. More recent work by Seo (2004) and Kapetanios et al. (2006) and Nedeljkovic (2008) proposed LM-type tests for nonlinear adjustment in the context of a Smooth Transition VECM (STVECM). A central analytical issue with tests for the unit root hypothesis and the nonlinear adjustment as well as nonlinear cointegration is the fact that a Davies (1987) problem typically occurs in these settings; i.e., parameter(s) can be unidentified under the null hypothesis of no-unit root, no-cointegration or no-nonlinear adjustment. The two solutions that have been proposed to tackle this issue are (i) using a Taylor series expansion approach and (ii) optimizing a test statistic over the parameters that are unidentified under the null hypothesis.

Within the context of ST-VECMs, Seo (2004) considers testing for nonlinearity in a system with a single cointegrating vector and unique transition function across equations. Nedeljkovic (2008) on the other hand extends Seo (2004)'s test to a more general model with multiple cointegrating vectors and multiple transitions across equations. Both tests are based on the score functions evaluated at the null hypothesis that no nonlinear adjustment to the (linear) cointegrating relationship hence rely on the restricted linear VECM. The limit distribution of Seo (2004)s and Nedeljkovic (2008)'s statistics depend on various nuisance parameters, including the short- and long-run variance parameters, and hence they propose to use various bootstrap procedures for obtaining critical values. Both of these tests are also based on optimizing over a fixed grid for the unidentified parameter(s) under the null of no-nonlinear adjustment. Kapetanious et al., (2006) on the other hand, propose two tests based on Taylor series approximations; one for no-coinegration and one for no-nonlinear adjustment in a ST-VECM. Their test has the advantage that it does not require bootstrap methods and the asymptotic distribution is nuisance-parameter free. The drawback of their testing approach is that in their setting, the Taylor series expansion will not become more accurate in some asymptotic sense and hence the inference may be affected by the approximation errors depending on the distance of data generating process from the local alternative.

In this paper, we propose a test statistic that optimizes over the parameter(s) that are unidentified under the null and is asymptotically pivotal. In the spirit of Kapetanious et al. (2006), we suggest a sup-Wald statistic that tests the null hypothesis that there is no linear and nonlinear adjustment in the vector error correction model. Therefore, we view our paper as the natural follow-up to Kapetanious et al. (2006)'s tests in the sense that instead of approximating the nonlinearity, we optimize over the unidentified parameter space and derive the asymptotic distributions of tests which are non-standard but nuisance parameter free within the same ST-VECM framework. Therefore, we compute and provide the asymptotic critical values by simulations. One advantage of our testing approach is it is easy to implement as it relies on Least Squares and does not require bootstrapping as in Seo (2004) or Nedeljkovic (2008). Moreover, our test avoids the potential approximation error the Kapetanious et al. (2006) approach may have. Monte Carlo simulations show the proposed test has good size and power properties in finite samples. Simulations also show that the proposed test perform considerably well compared to tests of Kapetanious et al. (2006) and tests of Johansen (1995) and Horwath and Watson (1994) in terms of size and power in small samples. An empirical application to monetary model of exchange rate determination for six major currencies reveals considerable evidence of cointegration and nonlinear adjustment that can be characterized by the past deviations from the long-run equilibrium as well as past inflation differential between the countries.

Section 2, discusses the smooth transition vector error correction model we consider and develops our testing procedure within this model. In section 3, we discuss our simulation evidence on the size and power performance of alternative tests. Application is discussed in section 4 and section 5 concludes our paper. The proof of main result is given in the Appendix.

2 Nonlinear Smooth Transition Error Correction and Testing

Following Kapetanios et al. (2006) in this paper we analyze at most one conditional cointegrating relationship between the scalar variable, y_t and the $k-$dimesnional vector variable, $\boldsymbol{x}_t$. Our focus is on the conditional modeling of the scalar variable y_t given the $k-$vectors $\boldsymbol{x}_t$ and the past values of $\boldsymbol{z}_t = (y_t, \boldsymbol{x}_t^{'})'$. For this purpose consider the following conditional smooth transition error correction model (STECM) for the scalar Δy_t and the marginal vector autoregressive (VAR) model for the $\Delta \boldsymbol{x}_t$

$$\Delta y_t = \phi e_{t-1} + \phi^* e_{t-1} F(\gamma,\, v_{t-d}) + \boldsymbol{\omega}' \Delta \boldsymbol{x}_t + \sum_{i=1}^{p} \boldsymbol{\psi}_i^{'} \Delta \boldsymbol{z}_{t-i} + u_t \tag{1}$$

$$\Delta \boldsymbol{x}_t = \sum_{i=1}^{p} \boldsymbol{\Gamma}_i^{'} \Delta \boldsymbol{z}_{t-i} + \boldsymbol{\eta}_t \tag{2}$$

where $u_t \sim iid(0, \sigma_u^2)$, with $\sigma_u^2 = \sigma_{yy} - \sigma_{y\boldsymbol{x}} \boldsymbol{\Sigma}_{xx}^{-1} \sigma_{\boldsymbol{x}y}$ (where now σ_{yy}, σxy and $\Sigma_{\boldsymbol{xx}}$ are the variance of scalar variable y, covariance vector between y and $\boldsymbol{x}$ and the variance-covariance matrix for $\boldsymbol{x}$ respectively), $e_{t-1} = y_{t-1} - \boldsymbol{\beta}' \boldsymbol{x}_{t-1}$ with $\boldsymbol{\beta}$ is the $k \times 1$ cointegration vector. The STECM model in Eqns. (1) and (2) follow from a general nonlinear VEC model for the vector $\boldsymbol{z}_t = (y_t, \boldsymbol{x}_t^{'})'$ under some iid and moment assumptions on the error term, given initial values for $\boldsymbol{z}_t$, and the weak exogenity of $\boldsymbol{x}_t$. Note also that u_t is uncorrelated with $\boldsymbol{\eta}_t$ by construction as discussed in KSS and further we assume that there is no cointegration among the $k-$vector of $I(1)$ variables $\boldsymbol{x}_t$. For brevity, we refer reader to KSS for the details of model in Eqns. (1) and (2). Note also that by construction the model in (1) and (2) imply that the process $\boldsymbol{x}_t$ is weakly exogenous. Following KSS we further assume that Δy_t does not Granger cause $\Delta \boldsymbol{x}_t$ in (2) and thus coefficients corresponding to Δy_t in $\boldsymbol{\Gamma}_i$ are zero. Under these assumptions the multivariate invariance principle

$$T^{-1/2}\sum_{t=1}^{[Tr]}(u_t,\, \boldsymbol{\eta}_t^{'})^{'} \Rightarrow (\sigma_u \boldsymbol{W}(r),\, \boldsymbol{\Sigma}_{xx}^{1/2}\boldsymbol{W}_x(r)^{'})^{'},$$

where $[Tr]$ is the integer part of Tr, $\Rightarrow$ denotes a weak convergence and $W(r)$ and $\boldsymbol{W}_x(r)$ defined on $r \in [0,\, 1]$, are independent scalar and $k-$vector standard Brownian motions, holds.

Stability properties of STVECM in (1) have been studied by Saikkonen (2005, 2008). Saikkonen (2005, 2008) shows that under relatively mild assumptions on boundedness and asymptotic domination of the nonlinear function $F(.)$ and existence of moments of the vector random process $(u_t, \boldsymbol{\eta}_t)$, the stochastic processes $\{\Delta \boldsymbol{z}_t\}$ and $\{e_{t-1}\}$ are geometrically ergodic and hence there exist a set of initial values of $(z_p,\, \cdots,\, z_0)$ such that the process $\Delta \boldsymbol{z}_t$ is strictly stationary and absolutely regular with geometrically decaying mixing numbers. As shown in KSS global stationarity of the STVECM will be satisfied provided that $\phi + \phi^* < 0$.

Two commonly used transition functions which we consider in this paper, namely the exponential and the logistic transition functions clearly satisfy the conditions required for the strict stationarity of the model in (1). The exponential and the logistic forms of adjustments in the STVECM are characterized by the following exponential,

$$F(\gamma, \mu, v_t) = 1 - \exp\left(-\gamma(v_{t-d})^2\right), \tag{3}$$

and logistic,

$$F(\gamma,\, c,\, v_t) = 1/\left[1 + \exp\left(-\gamma(v_{t-d})\right)\right], \tag{4}$$

functions respectively. In Eqns. (3) and (4) v_{t-d} is the transition variable, d is the delay integer valued-parameter γ is a slope parameter (or the transition parameter) and c is a location parameter (or the threshold parameter). The parameter restriction $\gamma > 0$ is an identifying restriction. The transition variable we consider in this paper includes the deviation from the cointegration (i.e. e_{t-l} with $l \geq 1$) or it can be any exogenous or predetermined variable dictated by the economic theory.

The conditional STVECM given in Eqn. (1) with the transition function in Eqn. (3) implies that for sufficiently large enough positive and negative lagged deviations of v_t the the underlying system will tend to the long-run equilibrium and the adjustment is symmetric across negative and positive deviations. On the other hand, the STVECM with the logistic transition function implies asymmetric adjustment in the sense that adjustment dynamics may differ for positive and negative deviations of the transition variable. In both cases, the speed of adjustment is governed by the magnitude of the slope (transition) parameter.

We consider the hypotheses of no-cointegration (or no linear and nonlinear adjustment) under STVECM. The hypothesis of no-cointegration is given by $H_0^1 = \phi = \phi^* = 0$ against the alternative hypothesis that the null is not true. Observe that under the null of no cointegration, the model in Eqn. (1) implies that the cointegration relation given by $e_{t-1} = y_{t-1} - \boldsymbol{\beta}'\boldsymbol{x}_{t-1}$ and hence the cointegrating vector (β is not identified. Moreover, the parameter that governs the nonlinear component of the ST-VECM, namely γ is not identified under this null either. To avoid the nuisance parameters problem under the null hypothesis, KSS, Chi and Saikkonen (2004) approximates the nonlinear transition function around $\gamma = 0$. In this paper, we take an alternative route and optimize the associated Wald statistic over the unidentified parameter space for γ. As for the unidentified cointegration vector β, following Engle and Granger (1987) and KSS, we take a pragmatic residual-based two-step approach and obtain the residuals, $\hat{e}_t = y_t - \hat{\beta}'\boldsymbol{x}_t$ with $\hat{\beta}$ being the ordinary least squares (OLS) estimate of β when the cointegration vector is unknown.[1] On the other hand, we take the error process $e_t = y_t - \beta' \boldsymbol{x}_t$ where β is the pre-specified cointegrating vector when the cointegrating vector is presumed to be known and implied by an economic theory.

If the unidentified parameter γ were known, the usual Wald test statistic $W(\gamma)$ for a given γ can be obtained as usual from Eqn. (1). To overcome the identification problem under the null of no-cointegration, we suggest to optimize the $W(\gamma)$ over a fixed parameter space of γ that is normalized by the sample standard deviation of the transition variable v_{t-d}. In other words, we suggest to use

$$\sup_{\gamma\in\Gamma_T} W(\gamma) = \sup_{\gamma\in\Gamma_T} \left\{\widehat{\theta}(\gamma)' R \left(\hat{\sigma}^2(\gamma) R V(\gamma) R\right)^{-1} R' \widehat{\theta}(\gamma)\right\} \tag{5}$$

where R is the selector matrix $R = (\mathbf{0}, I_{k+(k+1)\times p})$, $\widehat{\theta}(\gamma) = \left(\hat{\phi}(\gamma),\, \hat{\phi}^*(\gamma),\, \hat{\boldsymbol{\omega}}(\gamma)',\, \hat{\boldsymbol{\psi}}(\gamma)'\right)$ is the $2 + k + (k+1) \times p$ parameter vector, $\hat{\sigma}^2(\gamma)$ is the least squares estimate of $\sigma^2(\gamma)$, that is $\hat{\sigma}^2(\gamma) = T^{-1} \sum_{t=1}^{T} (\Delta y_t - \hat{\phi} e_{t-1} - \hat{\phi}^* e_{t-1} F(\gamma,\, v_t) - \hat{\boldsymbol{\omega}}' \Delta \boldsymbol{x}_t - \sum_{i=1}^{p} \hat{\boldsymbol{\psi}}_i \Delta \boldsymbol{z}_{t-i})^2$ and $V(\gamma) = \left((e_t,\, e_t F(\gamma,\, v_{t-d}))',\, \Delta \boldsymbol{x}_t',\, \Delta \boldsymbol{z}_t'\right)' \left((e_t,\, e_t F(\gamma,\, v_{t-d}))',\, \Delta \boldsymbol{x}_t',\, \Delta \boldsymbol{z}_t'\right)$.

The parameter space $\Gamma_T = (\underline{\gamma},\, \overline{\gamma})$ is assumed to be a compact space in real line. Note that if v_t is a stationary process, we set $\Gamma_T = \left[\frac{1}{100 s_{v,T}},\, \frac{100}{s_{v,T}}\right]$ and if v_t has a unit root (this would be the case under the null of no-cointegration if $v_t = e_{t-1-d}$ for example, then we set $\Gamma_T = \left[\frac{1}{100 (s_{v,T})/\sqrt{T}},\, \frac{100}{(s_{v,T})/\sqrt{T}}\right]$ (with $s_{v,T}$ being the standard error of v_t). This choice of parameter space for the grid search ensures that under the null Γ_T has a well-defined limit as $T \to \infty$ with the limit given by $\Gamma = \left[\underline{\gamma},\, \overline{\gamma}\right] = \left[\frac{1}{100\sigma_v},\, \frac{100}{\sigma_v}\right] \subseteq R^+$. Note also that our choice for Γ_T depends not only on the sample size but also the scale of the transition variable[2] and as $T \to \infty$ it converges to a well-defined compact set in real line. This is

[1] Note that under the null the VECM estimates do not depend on γ and $\sqrt{T}(\hat{\beta} - \beta_0) = o_p$.

[2] Since the unidentified parameter space for γ is infinite, making the parameter space a function of the scale of the transition variable should ensure the space is dense in the unidentified parameter as emphasized by Hansen (1996).

in contrast to the choice used in Seo (2004) and Nedeljkovic (2008) who optimize their LM-type statistic over a constant parameter space.

Theorem 1 *Consider the conditional STECVM in (1) under the assumptions discussed above. Under the null hypothesis of no cointegration, the supW statistic defined by (9), has the following asymptotic distribution:*

$$\begin{aligned} &\sup_{\gamma\in\Gamma_T} W(\gamma) \Rightarrow \sup_{\gamma\in\Gamma} \left[\begin{array}{cc} \int_0^1 W(r)dr & \lambda\int_0^1 W(r)dV(r,\gamma) \end{array} \right] \\ &\times \left[\begin{array}{cc} \int_0^1 W(r)dW(r) & \lambda\int_0^1 W^2(r)dV(r,\gamma) \\ \lambda\int_0^1 W^2(r)dV(r,\gamma) & \kappa\int_0^1 W^2(r)dr \end{array} \right]^{-1} \left[\begin{array}{c} \int_0^1 W(r)dW(r) \\ \lambda\int_0^1 W(r)dV(r,\gamma) \end{array} \right] \end{aligned} \tag{6}$$

where $(W(r), V(r,\gamma))$ is a bivariate Brownian Motion process for a each given γ with covariance matrix[3]

$$\Sigma(\gamma) = \begin{pmatrix} 1 & \sigma_{12}(\gamma) \\ \sigma_{12}(\gamma) & 1 \end{pmatrix} \tag{7}$$

and

$$\sigma_{12}(\gamma) = EF(\gamma v_{s-d})/[EF(\gamma v_{s-d})^2]^{1/2}. \tag{8}$$

Proof: See Appendix.

This theorem shows that the asymptotic distribution of the test statistic has the χ^2 form and is nonstandard, but nuisance parameter free and depends only on the limit parameter space Γ which in turn is a fixed interval that is scaled by the standard error of the transition variable v_t. Note however that depending on the functional form of the nonlinear transition function and the transition variable the distribution of the test

[3]Strictly speaking, a bivariate Brownian Motion only depend upon one index, namely time. We should emphasize that for a given γ value $(W(r), V(r,\gamma))$ is a bivariate Brownian Motion and hence the notation is used to emphasize dependence on a given γ.

statistic will be different. In other words, the asymptotic distribution and hence the critical values of the *supW* test will be different depending on if the STVECM is given by equations (1) and (3) or equations (1) and (4).

Following KSS, accommodation of deterministic components in the cointegrating relation, $e_t = y_t - \beta' \boldsymbol{x}_t$, can be achieved by including an intercept and an intercept and a linear trend, or simply by using the residuals from the first-stage regression where the data is either demeaned or de-trended. In other words the cointegration relation is now given by $e_t^* = y_t^* - \beta' \boldsymbol{x}_t^*$ and $e_t^+ = y_t^+ - \beta' \boldsymbol{x}_t^+$ where the superscripts $*$ and $+$ indicate the demeaned and the detrended data respectively. Following the arguments in KSS, we should observe that for the regression with a nonzero intercept, the asymptotic distribution of the *supW* test is the same as in Theorem 1, except that $W(r)$ and $V(r, \gamma)$ are replaced with demeaned Brownian motions, denoted $\tilde{W}(r)$ and $\tilde{V}(r, \gamma)$. In a similar fashion, for the regression with a nonzero intercept and linear trend in the cointegrating relation, the associated asymptotic distribution is given as in Theorem 1 with the exception that $W(r)$ and $V(r, \gamma)$ are replaced by the demeaned and detrended versions, denoted $\hat{W}(r)$ and $\hat{V}(r, \gamma)$.

Asymptotic critical values of the $supW^E$ and $supW^L$ statistics for the preceding three cases are tabulated under various transition variables in via stochastic simulations with $T = 1,000$ and $50,000$ simulations Table 1. The first panel in Table 1 gives the critical values when the cointegrating vector (β) is assumed to be known. In the second panel of Table 1, we report asymptotic critical values when the cointegrating vector is estimated by OLS in a first-stage regression as in Engle and Granger (1987). Since in this case, the distribution of the tests depend on the sampling uncertainty associated with the estimation of the coingeration vector in the first stage, critical values are reported for $k = 1, \cdots, 5$. Finally the last panel of Table 5 reports the asymptotic critical values when the transition variable is an exogenous variable which follows a stationary $AR(1)$

process.

3 Finite-Sample Properties of tests

In this section we present and discuss small-sample size and power properties of our proposed *supW* test via Monte-Carlo experiments. We also investigate performance of our proposed test two other closely related tests, namely the F^*_{NEC} statitic proposed by KSS and the linear cointegration test of Horwath and Watson (1994).[4]

We follow KSS and generate the data from the following bivariate ECM:

$$\Delta y_t = \phi e_{t-1} + \phi^* e_{t-1} F(\gamma, v_t) + \omega \Delta x_t + u_t,$$

$$\Delta x_t = \eta_t, \;\; e_t = y_t - \beta x_t,$$

$$\begin{bmatrix} u_t \\ \eta_t \end{bmatrix} \sim iid\mathcal{N}\left(\mathbf{0}, \begin{bmatrix} \sigma_u^2 & 0 \\ 0 & \sigma_\eta^2 \end{bmatrix}\right),$$

where we set $\beta = 1$ and $\sigma_u^2 = 1$. Under the null of no cointegration we set $\phi = \phi^* = 0$ whereas we consider parameter values for $\gamma = 0.01,\ 0.1,\ 1.0,\ 10$, $\phi = -0.1,\ 0.0,\ 0.1$, $\phi^* = -1,\ -0.5,\ -0.3,\ -0.1$ and $\omega = 0.5,\ 1$ under the alternative. To investigate the impact of signal-to-noise ratios we consider $\sigma_\eta^2 = 1,\ 4$. We examine the size and power properties of tests under various setups for the transition variable. Specifically, we consider $v_t = e_{t-1}$ with the cointegrating vector is known ($\beta = 1$) and unknown and estimated by OLS. Moreover we also consider that the transition variable is given by an exogenous variable $v_t = z_{t-1}$ where z_t follows an $AR(1)$ process with AR parameter ρ is set to

[4]Note also that we do not consider the tests of Seo (2004) and Nedeljkovic (2008) as these tests require bootstrapping the critical values and hence differ from our test. However, it is desirable would be useful to undertake a careful investigation of these tests under various data generating processes as all of these tests are consistent against the alternative models. This however is beyond the scope of this paper.

$\rho = 0.5$, 0.9, 0.95, 1 therefore covering various degree of persistence. To conserve space we report results on the size of alternative tests for the demeaned data with sample size $T = 100$ in Table 2. Inspection of the Table reveals no major size distortions for any of the alternative tests. Therefore in the following we study power of the tests by using asymptotic critical values.

Results from the simulations on the power performance of alternative tests are displayed in Tables 3 through 6. In each table first panel gives the power of the tests when the transition function is assumed to be exponential while second panel report results for the logistic transition function. In order to conserve space, we provide results when the transition variable is the lagged residuals from the cointegrating relationship (i.e., $\hat{e_{t-1}}$). Findings when the transition variables are the cointegrating relation (i.e. e_{t-1}) and an exogenous variable z_{t-1} are not displayed. Overall results are qualitatively similar and can be obtained upon request.

Careful inspection of the displayed results in Tables 3 through 6 clearly shows that our $supW$ tests outperform alternatives in majority of the cases considered. Findings show that both $supW^E$ and $supW^L$ tests perform considerably well compared to alternatives when the nonlinear adjustment in the VECM is given by the exponential and logistic functions respectively. Results also illustrate that each of these tests perform well even if $supW^E$ and $supW^L$ are used when the adjustment dynamics is incorrectly specified (i.e. when say the transition function is exponential form and $supW^L$ is used or the other way around). Reported results show that as the slope of the transition increases (i.e. as γ increases) the power of all tests tend to increase with substantial power gains obtained for $supW$ tests. Even when $\phi^* = 0$, the power of our tests are considerably higher than the alternatives and the power gains increases as ϕ^* moves away from zero in both directions. We should also note that power gains are asymmetric in the sense that all tests tend to show more power gains for negative values of ϕ^* than the positive values of ϕ^*. This makes sense as for a given value of ϕ the sum $\phi + \phi^*$ becomes more negative and hence the process becomes less persistent under the alternative.

Reported simulation results reveal that generally speaking tests based on nonlinear VECMs outperform the tests based on linear VECMs. In other words, both test of KSS and *supW* tests perform better than the Johansen (1991) and Horwath and Watson (1994) tests. This shows that considering tests that are designed specifically for the nonlinear VECMs may prove useful in applications as available tests based on linear VECMs may not detect cointegration when there is nonlinear adjustment in the VECM. Results also reveal that usually test of Horwath and Watson (1994) performs better than the Johansen (1991) test in the presence of nonlinear dynamics. Reported results also show that *supW* tests show considerable power gains over the KSS test as expected. This is partly because KSS test approximates the nonlinear adjustment by taking a Taylor series expansion of nonlinearity and hence may have power problems especially when the degree of nonlinear adjustment gets more acute.

4 Empirical Application: Exchange Rates and Monetary Fundamentals

Even though theories of the exchange rate determination imply that the exchange rate is determined by macroeconomic fundamental variables such as relative money supplies and incomes between two countries, the empirical evidence on the existence of the relationship and/or predictability of the future changes in exchange rate on the basis of fundamentals is still scarce (Cheung, Chinn and Garcia-Pasqual, 2005). Cointegration analysis based on linear relationships between exchange rate and fundamentals has been one of the most frequently used methods in the empirical analysis. One possibility is that the deviations of the exchange rate from the level determined by the fundamentals may be governed by a nonlinear adjustment process which is difficult to capture using linear methods. Therefore, even though a stable long-run relationship may exist, failure to correctly model the short-run adjustment dynamics may suggest non-existence of such a relation between exchange rate and fundamentals.

It is well-known that different types of nonlinearity may arise in the foreign exchange market due to heterogeneity of agents. Overall, when the exchange rate is close to its fundamental value the influence of technical analysis and other types of trading strategies dominates the market. Conversely, when the exchange rate becomes increasingly misaligned with the fundamentals, the pressure from both policy makers and agents for returning the exchange rate to the neighborhood of the long-run level becomes stronger and eventually dominates the market.

Taylor and Peel (2000) and Kilian and Taylor (2003) found significant evidence of nonlinearity in the deviations from the monetary fundamental equilibrium, showing that the exchange rate follows near-unit process for small deviations (implying lack of cointegration found in previous studies), but fast mean-reversion for large departures from the long-run equilibrium. They used linearity tests and fitted ESTAR models to the deviations of exchange rate from fundamentals. In this part of the paper, we investigate the presence of nonlinear adjustment in the context of STVECM which may provide useful insights on the existence of a stable long-run relationship and nonlinearity in the adjustment towards the long run equilibrium. The methodology developed in the paper may provide statistical evidence if the exchange rate and the fundamentals have a long-run relationship and if the adjustment to this long-run equilibrium is characterized by exponential and/or logistic type nonlinearities.

We consider a classical monetary model of exchange rate determination, which implies existence of the long-run relationship between exchange rate, money and output differentials of two countries. We consider two variables that can possibly characterize the nonlinear adjustment process in the monetary model, namely the level of deviation from the long-run equilibrium and the inflation differential between two countries. The motivation for the deviation of exchange rates from the fundamentals or the exchange rate level that is consistent with monetary fundamentals seems the natural variable which may characterize adjustment dynamics as also found by Taylor and Peel (2000). Motivation for the inflation differential comes from the empirical findings that suggest that inflationary environment and hence the inflation differential may be reflected on average proportionally in long-run nominal exchange rate depreciation (see, Coakley et al. 2005) during floating period.

Monthly data on the nominal exchange rate, monetary aggregate, industrial production and consumer prices for the United States (US), Canada, Denmark, Japan, Sweden, Switzerland, and the United Kingdom (UK) is collected from the IMF's International Financial Statistics for the period 1974-2008. The sample includes 420 observations for US, Denmark, Japan, Sweden and the UK. The data on money supply for Canada is available since March, 1975 (and hence 406 observations are available in the complete data set) and the industrial production data for Switzerland is available only from January 1995 (making 156 observations available for the analysis).

All series are expressed in logarithmic terms and they are first checked for a unit root, which, as expected, shows evidence of the nonstationarity. The VAR is then fit to the data, where the number of lags is computed using AIC and SIC criteria. Multivariate LM tests were also performed to ensure that VAR residuals are indeed white noise.[5] In Table 5, we report our findings from Johansen's (1995) Trace statistics (JOH) and Horwath and Watson (1994)'s (HW) tests. Johansen's test finds evidence of cointegration between

[5]The results for both unit root tests and residual tests are not reported in order to save space, but are available from the author.

the the exchange rate and monetary fundamentals for only Denmark. Horwath-Watson test shows that for three out of six countries there is evidence of cointegration between the Yen, Swiss Francs and UK Pound and US Dollar and respective money supply and output differentials with a cointegration vector of $(1, -1, -1)$. Horwath-Watson test also indicate presence of cointegration for Denmark, Japan, and Sweden with an unknown cointegration vector. Overall, cointegration tests designed within the context of linear VECM suggests some evidence of cointegration.

In Table 5, we report results from KSS and *supW* tests that take nonlinear adjustment in the STVECM discussed in this paper. In computing *supW* tests, we allow for the presence of both types of nonlinear adjustments (exponential and logistic) in our STVECM. We also consider three possible transition variables, namely the deviation from the cointegrating relationship when cointegration vector is pre-sepcified and is given by $(1, -1, -1)$, deviation from the cointegration relation where the cointegration vector is pre-estimated by OLS as in Engle and Granger (1987), and an exogenous variable namely the inflation differential between a given country and the US. KSS test shows evidence of cointegration for four out of six countries, namely Canada, Denmark, Japan and the UK at 10% significance level. $supW^E$ statistic shows evidence of cointegration with exponential form of adjustment dynamics for all currencies investigated at 5 or 10 percent levels with transition variable is given by the lagged deviations from the cointegration relationship. The same statistic shows evidence of cointegration with the inflation differential being the transition variable only for two countries however.

When the cointegration vector is not pre-specified, the $supW^E$ test still shows considerable evidence of cointegration (for five out of six countries). Interestingly, the evidence of cointegration with the inflation differential being the transition variable increases to four. Considering the logistic STVECM, $supW^L$ test shows evidence of cointegration for only two countries (namely Switzerland and the UK) with a known cointegration vector and the deviation from the cointegration characterizes the nonlinear adjustment. Not restricting the cointegration vector provides evidence of cointegration for Yen, Swedish Kroner and the UK Pound. Using the inflation differential as the transition variable with logistic type of nonlinearity the $supW^L$ test finds evidence of cointegration for Canadian Dollar, Swedish Kroner and the UK Pound with the known cointegration vector. Not restricting the cointegration vector adds Danish Kroner to the list. Careful inspection of the results in Table 5 shows that KSS and $supW$ tests show considerable evidence of cointegration with various delays and under different transition variables and transition functions. The strongest evidence is found from the $supW^E$ test with the deviation from the cointegrating relationship is the transition variable.

These findings conforms the empirical evidence reported in Taylor and Peel (2000) in a direct way. Moreover, the results imply that macro fundamentals may respond in a nonlinear manner to the disequilibrium in the long-run relationship from one to five months before, where the response in monetary and real variables depends on the magnitude of over/under pricing of the exchange rate and/or the difference in inflationary environment across countries over time. These links can be exploited further to investigate whether the evidence of nonlinearity can be used to improve the forecast of the exchange rate and, at the same time, whether this also helps in predicting future levels of fundamentals, adding to recently proposed theoretical approach of Engel and West (2005). These are left for future research however.

5 Conclusion

In this paper we have proposed a testing procedure for the presence of nonlinearity in the adjustment towards a cointegrating relationship in the context of a smooth transition VECM where adjustment can be characterized by exponential and logistic functions under various transition variables including the lagged deviations from the long-run stable relationship and an exogenous autoregressive process. We have considered STVECMs where the cointegration vector is both pre-specified and pre-estimated by OLS. The asymptotic distribution of the test statistic is shown to be non-standard but nuisance-free under the null of no-cointegration. Therefore, the critical values are obtained via simulations. Monte Carlo simulations show encouraging results in terms of size and power performance of proposed test. Simulations show that the proposed test performs as good as some alternatives and under various data generating scenarios outperforms some linear alternatives and the test of KSS which is suggested under the same setup as in our alternative model with exponential transition function.

But future work is required to incorporate heteroscedasticity and the broader volatility models, both in terms of the statistical representation of the nonlinear system and for testing against nonlinearity in such framework. Another possible extension is to allow for the richer dependence structure in the errors of the process. This does not represent a serious deficiency of the current testing procedure since choosing the appropriate VAR order will "whiten" the errors. However, once the progress is made in terms of the Markovian representation of the system under such an assumption, the changes in the testing procedure would be straightforward using the battery of asymptotic results for dependent (but not necessarily absolutely regular with geometrically decaying mixing numbers) processes. The robust type of tests following White (1994) could also be considered in such framework. Relaxing the assumption of the known number of cointegrating vectors and testing directly for it is another fruitful area of research.

Empirical application reported in the paper showed that controlling for the nonlinear adjustment in testing for the cointegration may prove useful and can provide insights in our understanding of the disconnect puzzle in international finance. Results showed considerable evidence on the cointegration between exchange rates and monetary fundamentals especially when the nonlinear adjustment to the long-run equilibrium is taken into consideration. Therefore, future research can benefit from the testing procedure suggested in this paper in other applications.

Appendix: Proof of Results

Asymptotic distribution of *supW* test under the null

We will first show pointwise convergence in distribution of the test statistic. Then to complete the proof, we show stochastic equicontinuity with respect to $\gamma \in \Gamma = (\underline{\gamma}, \overline{\gamma})$. Note that if v_t is a stationary process, we set $\Gamma_T = \left[\frac{1}{100 s_{v,T}}, \frac{100}{s_{v,T}}\right]$ and if v_t has a unit root (this would be the case under the null of no-cointegration if $v_t = e_{t-1-d}$ for example, then we set $\Gamma_T = \left[\frac{1}{100(s_{v,T})/\sqrt{T}}, \frac{100}{(s_{v,T})/\sqrt{T}}\right]$ (with $s_{v,T}$ being the standard error of v_t). This choice of parameter space for the grid search ensures that under the null Γ_T has a well-defined limit as $T \to \infty$ with the limit given by $\Gamma = \left[\underline{\gamma}, \overline{\gamma}\right] = \left[\frac{1}{100\sigma_v}, \frac{100}{\sigma_v}\right] \subseteq R^+$. Without any loss of generality we assume that we optimize over this compact parameter space $\Gamma = \left\{\gamma : \underline{\gamma} \leq \gamma \leq \overline{\gamma}\right\}$.

First note that the *supW* statistic given in Eqn. (5) can be written alternatively as

$$\sup_{\gamma \in \Gamma_T} W(\gamma) = \sup_{\gamma \in \Gamma_T} \frac{1}{2} \frac{\boldsymbol{u}' \boldsymbol{M} \boldsymbol{E}_{-1}(\gamma) \left(\boldsymbol{E}'_{-1}(\gamma) \boldsymbol{M} \boldsymbol{E}_{-1}(\gamma)\right)^{-1} \boldsymbol{E}'_{-1}(\gamma) \boldsymbol{M} \boldsymbol{u}}{\hat{\sigma}^2(\gamma)} \tag{9}$$

where $\boldsymbol{u} = (u_1, \cdots, u_T)'$, $\boldsymbol{M} = I_T - \boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}\boldsymbol{H}'$ is the $T \times T$ idempotent matrix, $\boldsymbol{H} = (\Delta \boldsymbol{X}, \Delta \boldsymbol{Z}_{-1} \cdots, \Delta \boldsymbol{Z}_{-p})$ is the $T \times (k + p(k+1))$ data matrix with $\Delta \boldsymbol{X} = (\Delta \boldsymbol{x}_1, \cdots, \Delta \boldsymbol{x}_T)'$, $\Delta \boldsymbol{Z}_{-i} = (\Delta \boldsymbol{z}_{1-i}, \cdots, \Delta \boldsymbol{z}_{T-i})'$ for $i = 1, \cdots, p$, $\boldsymbol{E}_{-1}(\gamma) = (\boldsymbol{e}_0, \cdots, \boldsymbol{e}_{T-1})'$ with $\boldsymbol{e}_t = (e_t, e_t F(\gamma, v_t))'$, and $\hat{\sigma}^2(\gamma)$ is the least squares estimate of $\sigma^2(\gamma)$.

Now, define for $0 \leq r \leq 1$,

$$W_T(r) = T^{-1/2} \sum_{s=1}^{[rT]} u_s / \sigma,$$

$$V_T(r, \gamma) = T^{-1/2} \sum_{s=1}^{[rT]} u_t F(\gamma, v_s) / \lambda,$$

where $[rT]$ is the largest integer less than or equal to rT, $\sigma^2 = E u_t^2$, and

$$\lambda^2 = \lim_{T \to \infty} E(T^{-1/2} \sum_{s=1}^{[T]} u_t F(\gamma, v_s))^2$$

$$= E[u_t F(\gamma, v_s)]^2$$

Note also $T^{-1/2} e_t \Longrightarrow \sigma \varpi W(r)$ where ϖ is given by the coefficients of autoregressive distributed lag representation of the STECM model under the null hypothesis (see Kapetanious et al. 2006, pp. 300 for details).

Now by the functional central limit theorem and by Lemma 2.1 of Park and Phillips

(2001) since $F(.)$ is a regular transformation, for any given γ, $(W_T(r), V_T(r,\gamma)) \Rightarrow (W(r), V(r,\gamma))$ for a bivariate Brownian motion process $(W(.), V(.,.))$ as in the theorem. Moreover since $F(v_s,\gamma)$ is a regular transformation of v_s we can apply Lemma 2.1 of Park and Phillips (2001) and by convergence to stochastic integrals (see for instance Davidson (1994, Theorem 30.13)) and by the FCLT (see for example, Davidson (1994, Theorem 29.11)) we obtain following results under the conditions of the theorem;

$$T^{-1}\sum_{t=1}^{T} e_{t-1}u_t \Longrightarrow \sigma\varpi \int_0^1 W(r)dr, \tag{10}$$

$$T^{-1}\sum_{t=1}^{T} e_{t-1}u_t F(\gamma,\, v_t) \Longrightarrow \varpi\lambda\sigma \int_0^1 W(r)dV(r,\gamma) + \Lambda(\gamma), \tag{11}$$

$$T^{-1}\sum_{t=1}^{T} e_{t-1}F(\gamma,\, v_t)h_t = T^{-1/2}\sum_{t=1}^{T}(T^{-1/2}e_{t-1})F(\gamma,\, v_s)h_t = O_p(1), \tag{12}$$

where

$$\Lambda(\gamma) = \lim_{T\to\infty} T^{-1}\sum_{t=1}^{T} E(e_{t-1}u_t F(\gamma, v_{t-d})) = 0,$$

$$T^{-2}\sum_{t=1}^{T} e_{t-1}^2 F(\gamma,\, v_t)^2 = T^{-1}\sum_{t=1}^{T}(\frac{e_{t-1}}{\sqrt{T}})^2 F(\gamma,\, v_t)^2 \Longrightarrow \kappa\sigma^2\varpi^2 \int_0^1 W(r)^2 dr. \tag{13}$$

and

$$T^{-2}\sum_{t=1}^{T} e_{t-1}^2 u_t^2 = T^{-1}\sum_{t=1}^{T}(\frac{e_{t-1}}{\sqrt{T}})^2 u_t^2 \Longrightarrow \sigma^2\varpi^2 \int_0^1 W(r)^2 dr. \tag{14}$$

$$T^{-2}\sum_{t=1}^{T} e_{t-1}^2 F(\gamma,\, v_t) = T^{-1}\sum_{t=1}^{T}(\frac{e_{t-1}}{\sqrt{T}})^2 F(\gamma,\, v_t) \Longrightarrow \sigma^2\varpi^2\lambda \int_0^1 W(r)^2 dV(r,\gamma). \tag{15}$$

where $\kappa = E(F(\gamma,\, v_t)^2)$. Using the results in Eqns. 10 through 15 and defining the scaling matrix $\boldsymbol{D}_T = diag(T^{-1},\, T^{-1})$ it can be seen that

$$\begin{aligned} \boldsymbol{D}_T\boldsymbol{E}_{-1}^{'}\boldsymbol{M}\boldsymbol{u} &= \boldsymbol{D}_T\boldsymbol{E}_{-1}^{'}\boldsymbol{u} + T^{-1/2}\left(\boldsymbol{D}_T\boldsymbol{E}_{-1}^{'}\boldsymbol{H}\right)\left(T^{-1}\boldsymbol{H}'\boldsymbol{H}\right)^{-1}\left(T^{-1/2}\boldsymbol{H}'\boldsymbol{u}\right) \\ &= \boldsymbol{D}_T\boldsymbol{E}_{-1}^{'}\boldsymbol{u} + o_p(1) \Longrightarrow \sigma\varpi \begin{bmatrix} \int_0^1 W(r)dr \\ \lambda(\gamma)\int_0^1 W(r)dV(r,\,\gamma) \end{bmatrix}, \end{aligned} \tag{16}$$

$$\boldsymbol{D}_T\boldsymbol{E}_{-1}^{'}\boldsymbol{M}\boldsymbol{E}_{-1}\boldsymbol{D}_T = \boldsymbol{D}_T\boldsymbol{E}_{-1}^{'}\boldsymbol{E}_{-1}\boldsymbol{D}_T + T^{-1}\left(\boldsymbol{D}_T\boldsymbol{E}_{-1}^{'}\boldsymbol{H}\right)\left(T^{-1}\boldsymbol{H}^{'}\boldsymbol{H}\right)^{-1}\left(\boldsymbol{H}^{'}\boldsymbol{E}_{-1}\boldsymbol{D}_T\right)$$
$$= \boldsymbol{D}_T\boldsymbol{E}_{-1}^{'}\boldsymbol{E}_{-1}\boldsymbol{D}_T + o_p(1) \Longrightarrow \begin{bmatrix} \sigma^2\varpi^2\int_0^1 W(r)^2dr & \sigma^2\varpi^2\lambda\int_0^1 W(r)^2dV(r,\gamma) \\ \sigma^2\varpi^2\lambda\int_0^1 W(r)^2dV(r,\gamma) & \sigma^2\varpi^2\kappa(\gamma)\int_0^1 W(r)^2dr \end{bmatrix}. \tag{17}$$

$$\widehat{\sigma^2(\gamma)} = T^{-1}\sum_{t=1}^{T}\left(u_t - \hat{\phi}e_{t-1} - \hat{\phi}^{*}e_{t-1}F(\gamma,\, v_t) - (\hat{\boldsymbol{\omega}}-\boldsymbol{\omega})^{'}\Delta\boldsymbol{x}_t - \sum_{i=1}^{p}(\hat{\boldsymbol{\psi}}-\boldsymbol{\psi})^{'}\Delta\boldsymbol{z}_t\right)^2$$

It is easy to show that under null,

$$\widehat{\sigma}^2(\gamma) = T^{-1}\sum_{t=1}^{T}\left(u_t - \hat{\phi}e_{t-1} - \hat{\phi}^{*}e_{t-1}F(\gamma,\, v_t) - (\hat{\boldsymbol{\omega}}-\boldsymbol{\omega})^{'}\Delta\boldsymbol{x}_t - \sum_{i=1}^{p}(\hat{\boldsymbol{\psi}}-\boldsymbol{\psi})^{'}\Delta\boldsymbol{z}_t\right)^2$$
$$= T^{-1}\sum_{t=1}^{T}u_t^2 + o_p(1) \xrightarrow{p} \sigma^2, \tag{18}$$

where we have used $T\hat{\phi} = O_p(1)$, $T\hat{\phi}^{*} = O_p(1)$, $\sqrt{T}(\hat{\boldsymbol{\omega}}-\boldsymbol{\omega}) = O_p(1)$, and $\sqrt{T}(\hat{\boldsymbol{\psi}}_i-\boldsymbol{\psi}_i) = O_p(1)$. Using Eqns. 10, 11 and 12 in Eqn. (9), we obtain,

$$\sup_{\gamma\in\Gamma_T} = \sup_{\gamma\in\Gamma}\begin{bmatrix} \int_0^1 W(r)dr & \lambda\int_0^1 W(r)dV(r,\,\gamma) \end{bmatrix}$$
$$\times\begin{bmatrix} \int_0^1 W(r)dW(r) & \lambda\int_0^1 W^2(r)dV(r,\,\gamma) \\ \lambda\int_0^1 W^2(r)dV(r,\,\gamma) & \kappa\int_0^1 W^2(r)dr \end{bmatrix}^{-1}\begin{bmatrix} \int_0^1 W(r)dW(r) \\ \lambda\int_0^1 W(r)dV(r,\,\gamma) \end{bmatrix} \tag{19}$$

To show stochastic equicontinuity, note that because

$$\sup_{\gamma\in\Gamma}|\hat{\sigma^2}(\gamma)-\sigma^2| \xrightarrow{p} 0.$$

and because for $F(\gamma, v_t) = 1-\exp(-\gamma v_t^2)$ we have

$$T^{-2}\sum_{t=1}^{T}e_{t-1}^2[1-\exp(-\gamma v_t^2)]^2 \geq T^{-2}\sum_{t=1}^{T}e_{t-1}^2[1-\exp(-\underline{\gamma}v_t^2)]^2$$

and for the logistic transition function $F(\gamma, v_t) = \frac{1}{1+\exp(-\gamma v_t)}$, we have

$$T^{-2}\sum_{t=1}^{T}e_{t-1}^2[\frac{1}{1+\exp(-\gamma v_t)}]^2 \geq T^{-2}\sum_{t=1}^{T}e_{t-1}^2[\frac{1}{1+\exp(-\underline{\gamma}v_t)}]^2,\ \text{for}\ v_t \geq 0,$$

and

$$T^{-2}\sum_{t=1}^{T}e_{t-1}^2[\frac{1}{1+\exp(-\gamma v_t)}]^2 \geq T^{-2}\sum_{t=1}^{T}e_{t-1}^2[\frac{1}{1+\exp(-\overline{\gamma}v_t)}]^2,\ \text{for}\ v_t < 0,$$

and because the last terms in each of the aforementioned cases converge in distribution to an almost surely positive limit, it suffices to prove the stochastic equicontinuity of[6]

$$D_T(\gamma) = T^{-1}\sum_{t=1}^{T} e_{t-1}u_t F(\gamma,\, v_t),$$

$$T^{-2}\sum_{t=1}^{T} e_{t-1}^2 F(\gamma,\, v_t)^2.$$

and of

$$T^{-2}\sum_{t=1}^{T} e_{t-1}^2 F(\gamma,\, v_t).$$

Stochastic equicontinuity of the last term, $T^{-2}\sum_{t=1}^{T} e_{t-1}^2 F(\gamma,\, v_t)$, when the transition function is exponential form follows because by a Taylor series expansion of order 1, for some mean value $\tilde{\gamma}$ on the line between γ and γ',

$$\begin{aligned}
&\sup_{\gamma\in\Gamma}\sup_{\gamma':|\gamma-\gamma'|<\delta} |T^{-2}\sum_{t=1}^{T} e_{t-1}^2[1-\exp(-\gamma v_t^2)] - T^{-2}\sum_{t=1}^{T} e_{t-1}^2[1-\exp(-\gamma' v_t^2)]| \\
&\le \max_{1\le t\le T}(T^{-1/2}e_t)^2 \sup_{\gamma\in\Gamma}\sup_{\gamma':|\gamma-\gamma'|<\delta} T^{-1}\sum_{t=1}^{T} |\gamma-\gamma'|\exp(-\tilde{\gamma}v_t^2)v_t^2 \\
&\le \delta \max_{1\le t\le T}(T^{-1/2}e_t)^2 T^{-1}\sum_{t=1}^{T}\exp(-\underline{\gamma}v_t^2)v_t^2 = \delta O_p(1),
\end{aligned}$$

where the $O_p(1)$ term does not depend on (γ, γ'). When the transition function is the logistic form then using the Taylor series expansion of order 1, for some mean value $\tilde{\gamma}$ on the line between γ and γ' we have

$$\begin{aligned}
&\sup_{\gamma\in\Gamma}\sup_{\gamma':|\gamma-\gamma'|<\delta} |T^{-2}\sum_{t=1}^{T} e_{t-1}^2[\frac{1}{1+\exp(-\gamma v_t)}] - T^{-2}\sum_{t=1}^{T} e_{t-1}^2[\frac{1}{1+\exp(-\gamma' v_t)}]| \\
&\le \max_{1\le t\le T}(T^{-1/2}e_t)^2 \sup_{\gamma\in\Gamma}\sup_{\gamma':|\gamma-\gamma'|<\delta} T^{-1}\sum_{t=1}^{T} |\gamma-\gamma'|F(\tilde{\gamma}',\, v_t)^2\exp(-\tilde{\gamma}v_t)|v_t| \\
&\le \delta \max_{1\le t\le T}(T^{-1/2}e_t)^2 T^{-1}\sum_{t=1}^{T} F(-\overline{\gamma}', v_t)^2\exp(-\underline{\gamma}v_t)|v_t| = \delta O_p(1),
\end{aligned}$$

where the $O_p(1)$ term does not depend on (γ, γ'). Similarly arguments can be used to

[6] Note also that the terms in Eqns. 10 and 15 do not depend on γ and the term in Eqn 10 is $O_p(1)$ and hence in the limit disappears.

show that the term $T^{-2}\sum_{t=1}^{T} e_{t-1}^2 F(\gamma,\, v_t)^2$ is also bounded in probability. To see this consider again a Taylor series expansion of order 1, for some mean value $\tilde{\gamma}$ on the line between γ and γ' when $F(\gamma,\, v_t)$ is given by the exponential function,

$$\sup_{\gamma\in\Gamma}\sup_{\gamma':|\gamma-\gamma'|<\delta}|T^{-2}\sum_{t=1}^{T} e_{t-1}^2[1-\exp(-\gamma v_t^2)]^2 - T^{-2}\sum_{t=1}^{T} e_{t-1}^2[1-\exp(-\gamma' v_t^2)]^2|$$

$$\leq \max_{1\leq t\leq T}(T^{-1/2}e_t)^2 \sup_{\gamma\in\Gamma}\sup_{\gamma':|\gamma-\gamma'|<\delta} T^{-1}\sum_{t=1}^{T}|\gamma-\gamma'|2[1-\exp(-\tilde{\gamma}' v_t^2)]\exp(-\tilde{\gamma} v_t^2)v_t^2$$

$$\leq \delta \max_{1\leq t\leq T}(T^{-1/2}e_t)^2 T^{-1}\sum_{t=1}^{T} 2[1-\exp(-\overline{\gamma}' u_{t-1}^2)]\exp(-\underline{\gamma} v_t^2)v_t^2 = \delta O_p(1),$$

where the $O_p(1)$ term does not depend on (γ,γ') again. One can similarly show the above results when the logistic function is the transition function and to conserve space we do not report the steps of the proof here. Now note that because $\max_{1\leq t\leq T} T^{-1/2}|e_t| = O_p(1)$, $D_T(\gamma)$ is asymptotically equivalent uniformly in γ as $K\to\infty$ to

$$D_T^K(\gamma) = T^{-1}\sum_{t=1}^{T} e_{t-1} I(T^{-1/2}|e_{t-1}|\leq K)u_t F(\gamma,\, v_t),$$

so it suffices to prove stochastic equicontinuity of $D_T^K(\gamma)$. To show this, note that the summands of $D_T^K(\gamma)$ are martingale differences, and therefore by the Burkholder inequality (see e.g. Hall and Heyde (1980, Theorem 2.10)),

$$E|D_T(\gamma)-D_T(\gamma')|^4$$

$$\leq C\left(T^{-2}\sum_{t=1}^{T} Ey_{t-1}^2 I(T^{-1/2}|e_{t-1}|\leq K)u_t^2(F(\gamma,\, v_t)^2 - F(\gamma',\, v_t)^2)\right)^2$$

$$\leq C(T^{-1}\sum_{t=1}^{T} K^2 Eu_t^2(F(\gamma,\, v_t)^2 - F(\gamma',\, v_t)^2))^2$$

which implies that for $F(\gamma,\, v_t) = 1-\exp(\gamma v_t^2)$,

$$\leq CK^2|\gamma-\gamma'|^2 Eu_t^2 Ev_t^2 2(1-\exp(-\overline{\gamma} v_t^2))\exp(-\underline{\gamma} v_t^2)$$

$$\leq C'|\gamma-\gamma'|^2.$$

Similarly for $F(\gamma,\, v_t) = \frac{1}{1+\exp(-\gamma v_t)}$,

$$\leq CK^2|\gamma-\gamma'|^2 Eu_t^2 Ev_t(\frac{1}{1+\exp(-\overline{\gamma} v_t)})^2\exp(-\underline{\gamma} v_t)$$

$$\leq C'|\gamma - \gamma'|^2.$$

Now by the the Theorem 15.5 of Billingsley (1968), it follows that $D_T(\gamma)$ is stochastically equicontinuous ("tight" in Billingsley's terminology). This completes the proof. $\square$

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Table 1: Asymptotic critical values for $supW^E$ and $supW^L$ tests

	Case 1			Case 2			Case 3		
	90%	95%	99%	90%	95%	99%	90%	95%	99%
Panel 1: Known CO vector with $v_t = e_{t-1}$									
$supW^E$	7.75	9.50	13.90	8.14	10.50	14.35	12.56	14.51	16.97
$supW^L$	6.04	7.76	11.62	7.25	9.40	12.28	9.65	12.10	15.55
Panel 2: Known CO vector with $v_t \sim AR(1)$ process									
$supW^E$	5.58	7.15	10.59	7.47	9.27	12.97	9.31	11.42	15.66
$supW^L$	4.92	6.34	9.51	6.78	8.58	11.92	8.71	10.63	14.77
Panel 3: Unknown CO vector with $v_t = \hat{e}_{t-1}$									
$supW^E$									
1	8.30	9.97	14.53	8.77	10.72	14.69	11.52	14.20	18.83
2	8.91	10.99	15.24	9.27	11.20	15.73	11.98	14.49	19.35
3	9.23	11.38	16.16	9.56	11.91	16.27	12.41	14.87	19.84
4	9.45	11.89	17.10	10.18	12.28	16.88	12.87	15.25	20.31
5	9.98	12.40	17.64	10.80	12.89	17.46	13.26	15.51	20.97
$supW^L$									
1	6.65	8.40	11.93	6.97	8.86	12.48	10.63	12.70	17.15
2	7.40	9.54	13.49	7.81	9.68	13.60	11.12	13.18	17.73
3	7.98	9.93	14.50	8.27	10.19	14.78	11.63	13.73	18.22
4	8.21	10.25	15.44	8.65	10.50	15.56	11.94	14.15	18.65
5	8.55	10.86	16.04	8.94	11.08	16.04	12.56	14.90	18.97
Panel 4: Unknown CO vector with $v_t \sim AR(1)$ process									
$supW^E$									
1	7.47	9.23	12.99	8.46	10.44	13.40	11.16	13.05	16.82
2	8.06	10.20	14.48	8.73	10.72	14.07	11.20	13.53	17.24
3	8.77	10.93	15.64	9.09	11.25	15.17	11.59	13.96	17.73
4	9.02	11.41	16.45	9.44	11.85	15.67	11.84	14.44	18.00
5	9.21	11.55	16.60	10.28	12.37	16.08	12.44	14.79	18.50
$supW^L$									
1	6.14	8.00	10.21	6.75	7.89	10.94	8.99	11.34	14.92
2	7.06	8.46	11.60	7.10	8.41	11.73	9.66	11.02	15.97
3	7.40	8.93	12.12	7.58	9.28	12.85	10.52	11.82	16.12
4	7.82	9.40	13.52	7.91	9.95	13.48	10.88	12.32	16.73
5	8.05	9.85	13.83	8.38	10.70	14.36	11.33	12.79	17.42

Table 2: Size of alternative tests for demeaned data ($T = 100$)

ω	σ_η^2	JOH	HW	KSS	$supW^E$	$supW^L$
0.5	1	0.066	0.066	0.042	0.056	0.054
0.5	4	0.062	0.063	0.044	0.055	0.053
1.0	1	0.059	0.057	0.045	0.053	0.046
1.0	4	0.055	0.054	0.048	0.047	0.042

Table 3: Power of alternative tests for the demeaned data: Transition variable is $v_t = e_{t-1}$ and sample size, $T = 100$, $\phi^* = 0.1$

Parameters				Exponential STVECM					Logistic STVECM				
ϕ	γ	ω	σ_η^2	JOH	HW	KSS	$supW^E$	$supW^L$	JOH	HW	KSS	$supW^E$	$supW^L$
-1.0	0.01	0.5	1	0.148	0.197	0.275	0.376	0.304	0.110	0.173	0.218	0.216	0.342
-1.0	0.01	0.5	4	0.288	0.318	0.495	0.671	0.550	0.335	0.315	0.257	0.274	0.490
-1.0	0.01	1.0	1	0.105	0.180	0.218	0.368	0.273	0.189	0.223	0.241	0.383	0.439
-1.0	0.01	1.0	4	0.166	0.233	0.335	0.404	0.356	0.200	0.247	0.334	0.392	0.460
-1.0	0.10	0.5	1	0.380	0.473	0.612	0.616	0.449	0.364	0.443	0.543	0.593	0.627
-1.0	0.10	0.5	4	0.385	0.450	0.703	0.884	0.704	0.375	0.437	0.526	0.594	0.790
-1.0	0.10	1.0	1	0.259	0.372	0.589	0.767	0.531	0.250	0.362	0.547	0.635	0.820
-1.0	0.10	1.0	4	0.287	0.446	0.637	0.770	0.628	0.300	0.440	0.633	0.725	0.864
-1.0	1.00	0.5	1	0.523	0.648	0.782	0.810	0.791	0.510	0.622	0.655	0.786	0.887
-1.0	1.00	0.5	4	0.598	0.634	0.761	0.927	0.767	0.590	0.623	0.740	0.827	0.929
-1.0	1.00	1.0	1	0.560	0.667	0.694	0.930	0.712	0.558	0.644	0.681	0.794	0.885
-1.0	1.00	1.0	4	0.582	0.693	0.815	0.973	0.789	0.560	0.674	0.783	0.857	0.918
-0.5	0.01	0.5	1	0.095	0.098	0.184	0.290	0.194	0.071	0.120	0.155	0.147	0.288
-0.5	0.01	0.5	4	0.189	0.205	0.316	0.410	0.322	0.196	0.244	0.208	0.211	0.424
-0.5	0.01	1.0	1	0.078	0.113	0.178	0.220	0.207	0.121	0.165	0.190	0.298	0.420
-0.5	0.01	1.0	4	0.100	0.162	0.240	0.300	0.284	0.142	0.190	0.207	0.321	0.443
-0.5	0.10	0.5	1	0.355	0.466	0.579	0.606	0.414	0.341	0.427	0.520	0.580	0.614
-0.5	0.10	0.5	4	0.362	0.452	0.684	0.871	0.657	0.358	0.430	0.517	0.588	0.773
-0.5	0.10	1.0	1	0.250	0.361	0.566	0.760	0.514	0.235	0.355	0.544	0.627	0.805
-0.5	0.10	1.0	4	0.281	0.440	0.623	0.764	0.607	0.283	0.436	0.618	0.710	0.849
-0.5	1.00	0.5	1	0.551	0.625	0.776	0.805	0.774	0.505	0.616	0.639	0.770	0.880
-0.5	1.00	0.5	4	0.579	0.630	0.760	0.916	0.760	0.568	0.620	0.732	0.813	0.924
-0.5	1.00	1.0	1	0.554	0.658	0.691	0.923	0.707	0.550	0.642	0.674	0.785	0.882
-0.5	1.00	1.0	4	0.580	0.685	0.809	0.962	0.781	0.552	0.667	0.775	0.852	0.909
-0.2	0.01	0.5	1	0.065	0.074	0.146	0.273	0.149	0.064	0.108	0.137	0.129	0.275
-0.2	0.01	0.5	4	0.112	0.161	0.271	0.389	0.303	0.167	0.216	0.201	0.206	0.411
-0.2	0.01	1.0	1	0.059	0.095	0.153	0.205	0.192	0.101	0.159	0.185	0.281	0.413
-0.2	0.01	1.0	4	0.085	0.110	0.219	0.296	0.265	0.129	0.168	0.198	0.314	0.426
-0.2	0.10	0.5	1	0.325	0.448	0.556	0.600	0.410	0.315	0.420	0.512	0.575	0.611
-0.2	0.10	0.5	4	0.338	0.440	0.667	0.848	0.639	0.352	0.422	0.510	0.583	0.760
-0.2	0.10	1.0	1	0.227	0.351	0.553	0.743	0.508	0.225	0.341	0.529	0.620	0.791
-0.2	0.10	1.0	4	0.266	0.433	0.616	0.750	0.600	0.276	0.425	0.609	0.700	0.835
-0.2	1.00	0.5	1	0.536	0.617	0.767	0.800	0.762	0.490	0.606	0.635	0.770	0.868
-0.2	1.00	0.5	4	0.565	0.623	0.759	0.911	0.754	0.556	0.612	0.730	0.808	0.900
-0.2	1.00	1.0	1	0.543	0.650	0.683	0.913	0.700	0.543	0.635	0.671	0.780	0.873
-0.2	1.00	1.0	4	0.568	0.678	0.800	0.948	0.768	0.550	0.661	0.770	0.840	0.883

Table 4: Power of alternative tests for the demeaned data: Transition function is exponential, transition variable is $v_t = e_{t-1}$ and sample size, $T = 100$, $\phi^* = 0.0$

Parameters				Exponential STVECM					Logistic STVECM				
ϕ	γ	ω	σ^2_η	JOH	HW	KSS	$supW^E$	$supW^L$	JOH	HW	KSS	$supW^E$	$supW^L$
-1.0	0.01	0.5	1	0.257	0.331	0.422	0.589	0.491	0.220	0.301	0.389	0.496	0.580
-1.0	0.01	0.5	4	0.670	0.772	0.840	0.910	0.835	0.484	0.630	0.780	0.823	0.860
-1.0	0.01	1.0	1	0.238	0.414	0.467	0.545	0.483	0.211	0.378	0.423	0.497	0.554
-1.0	0.01	1.0	4	0.255	0.456	0.494	0.521	0.490	0.240	0.443	0.478	0.503	0.556
-1.0	0.10	0.5	1	0.897	0.905	0.928	0.971	0.909	0.841	0.887	0.911	0.943	0.951
-1.0	0.10	0.5	4	0.925	0.944	0.953	0.980	0.941	0.908	0.925	0.939	0.952	0.964
-1.0	0.10	1.0	1	0.911	0.922	0.959	0.977	0.946	0.885	0.897	0.933	0.959	0.966
-1.0	0.10	1.0	4	0.910	0.926	0.947	0.968	0.953	0.880	0.903	0.939	0.963	0.972
-1.0	1.00	0.5	1	0.972	0.988	0.990	0.988	0.982	0.948	0.935	0.945	0.969	0.982
-1.0	1.00	0.5	4	0.965	0.980	0.983	1.000	0.978	0.955	0.964	0.970	1.000	1.000
-1.0	1.00	1.0	1	0.970	0.990	0.992	0.997	0.969	0.963	0.977	0.984	0.988	0.993
-1.0	1.00	1.0	4	0.986	0.977	0.990	0.994	0.985	0.983	0.975	0.987	0.988	0.991
-0.5	0.01	0.5	1	0.158	0.245	0.381	0.440	0.395	0.134	0.220	0.368	0.428	0.456
-0.5	0.01	0.5	4	0.380	0.514	0.669	0.655	0.576	0.368	0.339	0.650	0.608	0.643
-0.5	0.01	1.0	1	0.194	0.409	0.493	0.463	0.410	0.180	0.387	0.449	0.451	0.496
-0.5	0.01	1.0	4	0.213	0.421	0.536	0.500	0.414	0.202	0.393	0.512	0.484	0.498
-0.5	0.10	0.5	1	0.515	0.558	0.666	0.798	0.688	0.504	0.535	0.629	0.757	0.797
-0.5	0.10	0.5	4	0.566	0.563	0.739	0.806	0.697	0.533	0.540	0.713	0.764	0.825
-0.5	0.10	1.0	1	0.463	0.550	0.693	0.811	0.694	0.456	0.533	0.660	0.770	0.836
-0.5	0.10	1.0	4	0.488	0.570	0.744	0.820	0.697	0.463	0.459	0.715	0.787	0.840
-0.5	1.00	0.5	1	0.689	0.711	0.795	0.920	0.873	0.661	0.723	0.743	0.846	0.912
-0.5	1.00	0.5	4	0.710	0.736	0.812	0.931	0.877	0.683	0.749	0.778	0.895	0.955
-0.5	1.00	1.0	1	0.690	0.715	0.800	0.946	0.859	0.679	0.737	0.715	0.869	0.940
-0.5	1.00	1.0	4	0.700	0.744	0.815	0.980	0.884	0.697	0.780	0.812	0.890	0.929
-0.2	0.01	0.5	1	0.098	0.149	0.257	0.354	0.327	0.226	0.290	0.323	0.397	0.519
-0.2	0.01	0.5	4	0.211	0.315	0.263	0.422	0.386	0.271	0.321	0.538	0.585	0.581
-0.2	0.01	1.0	1	0.120	0.276	0.210	0.395	0.364	0.241	0.352	0.416	0.422	0.449
-0.2	0.01	1.0	4	0.130	0.289	0.268	0.446	0.383	0.250	0.372	0.460	0.456	0.455
-0.2	0.10	0.5	1	0.402	0.447	0.552	0.712	0.575	0.446	0.473	0.569	0.623	0.698
-0.2	0.10	0.5	4	0.440	0.488	0.593	0.733	0.669	0.457	0.479	0.605	0.652	0.729
-0.2	0.10	1.0	1	0.392	0.505	0.570	0.745	0.594	0.391	0.456	0.624	0.645	0.745
-0.2	0.10	1.0	4	0.379	0.515	0.580	0.780	0.638	0.389	0.430	0.650	0.705	0.768
-0.2	1.00	0.5	1	0.583	0.622	0.763	0.825	0.789	0.537	0.648	0.674	0.793	0.815
-0.2	1.00	0.5	4	0.616	0.660	0.778	0.915	0.796	0.565	0.663	0.750	0.822	0.840
-0.2	1.00	1.0	1	0.637	0.663	0.697	0.917	0.784	0.578	0.675	0.753	0.819	0.836
-0.2	1.00	1.0	4	0.648	0.684	0.755	0.953	0.810	0.626	0.682	0.788	0.863	0.890

Table 5: Results from Johansen's Trace and Horwath-Watson's Cointegration tests of exchange rate and monetary fundamentals

		Trace Statistics			*HW* Statistics	
Series	Lag	$r = 0$	$r = 1$	$r = 2$	CO-K	CO-U
Canadian Dollar	AIC, SIC (1)	20.85	7.77	2.95	6.55	19.37
Danish Kroner	AIC (6)	27.84	6.30	0.2.71	8.86	20.43^{*}
	SIC (2)	$39.20^{\ddagger}$	5.48	0.50	7.75	$32.18^{\ddagger}$
Japanese Yen	AIC (5)	$33.56^{\dagger}$	4.98	0.001	9.97^{*}	$27.43^{\ddagger}$
	SIC (3)	24.53	3.82	0.010	8.13	20.28^{*}
Swedish Kroner	AIC (2)	17.78	5.76	1.91	6.40	20.88^{*}
	SIC (1)	19.11	5.32	1.03	4.17	14.69
Swiss Francs	AIC, SIC (1)	19.68	5.10	0.48	10.95^{*}	16.21
UK Pound	AIC (3)	20.82	2.63	0.88	9.96^{*}	13.38
	SIC (2)	19.22	2.35	0.26	10.66^{*}	18.56

Table 6: Results from KSS and $supW$ tests of exchange rate and monetary fundamentals

		$KSS\,(d)$	$supW^E$				$supW^L$			
Series	Lag		CO-K		CO-U		CO-K		CO-U	
			e_{t-d} (d)	π_{t-d} (d)	$\hat{e}_{t-d}$ (d)	π_{t-d} (d)	e_{t-d} (d)	π_{t-d} (d)	$\hat{e}_{t-d}$ (d)	π_{t-d} (d)
Canadian Dollar	AIC, SIC (1)	13.71* (1)	14.32† (1)	11.65† (1)	8.39 (1)	10.90† (2)	5.49 (2)	10.69† (2)	8.37 (2)	9.36* (2)
Danish Kroner	AIC (6)	14.44* (6)	13.00† (6)	6.06 (5)	10.21* (6)	10.33* (1)	5.51 (5)	5.66 (2)	6.64 (2)	11.24† (2)
	SIC (2)	7.48(1)	5.76 (1)	6.12 (1)	12.28† (1)	8.73* (1)	3.21(1)	2.46 (2)	4.79 (1)	10.20* (2)
Japanese Yen	AIC (5)	12.94* (1)	9.51* (1)	5.79 (5)	12.53† (2)	4.13 (1)	3.01 (5)	6.15 (4)	11.70† (4)	5.16 (4)
	SIC (3)	8.95 (1)	8.62* (1)	4.04 (1)	12.14† (2)	3.97 (1)	2.39 (2)	1.87 (2)	8.17* (4)	3.04 (2)
Swedish Kroner	AIC (2)	5.27 (2)	8.29* (1)	3.85 (1)	11.91† (2)	8.80* (2)	5.10 (1)	9.69† (1)	9.50* (1)	9.62* (2)
	SIC (1)	5.14 (2)	8.17* (1)	3.20 (1)	11.60† (2)	6.93 (2)	2.70 (1)	8.83* (1)	8.85* (1)	8.66* (1)
Swiss Francs	AIC, SIC (1)	6.53 (2)	10.87† (2)	5.46 (1)	10.66* (2)	4.42 (2)	9.91† (2)	4.03 (1)	7.18 (1)	6.33 (1)
UK Pound	AIC (3)	5.88 (3)	12.88† (1)	10.01† (1)	9.10* (1)	10.95† (1)	18.48‡ (1)	11.55† (1)	17.05‡ (2)	11.84† (3)
	SIC (2)	13.42* (1)	10.66† (1)	11.01† (1)	8.53 (1)	11.23† (1)	28.39‡ (1)	18.97‡ (1)	26.14‡ (1)	6.68 (1)